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I. Solution by COOPER D. SCHMITT, A. M., Professor of Mathematics, University of Tennessee, Knoxville, Tenn.; P. S. BERG, A. M., Principal of Schools, Larimore, N. D.; J. OWEN MAHONEY, B. E., M. Sc., Graduate Fellow in Mathematics in Vanderbilt University, Lynnville, Tenn.; CHAS. A. JONES, Torrance, Miss.; and the PROPOSER.

$1+x^4=a+4ax+6ax^2+4ax^3+ax^4$.

$x^2+(1/x^2)=(a/x^2)+(4a/x)+6a+4ax+ax^2$.

$[x+(1/x)]^2=a[x+(1/x)]^2+4a[x+(1/x)]+4a+2 \ldots\ldots (1)$.

Let $x+(1/x)=y \ldots\ldots (2)$; $\therefore x^2-xy=1$.

$\therefore x=\frac{1}{2}[y\pm\sqrt{(y^2-4)}] \ldots\ldots (3)$. (2) in (1) gives

$y^2+[4a/(a-1)]y=-[(4a+2)/(a-1)]$.

$\therefore y=[\pm\sqrt{(2a+2)}-2a]/(a-1) \ldots\ldots (4)$. (4) in (3) gives

$x=[1/2(a-1)]\{-2a\pm\sqrt{[2a+2]}\pm\sqrt{[10a-2\pm4a\sqrt{(2a+2)}]}\}$.

II. Solution by J. SCHEFFER, A. M., Hagerstown, Md.; and F. M. McGAW, A. M., Professor of Mathemetics in Bordentown Military Institute, Bordentown, N. J.

Expanding and arranging we get, dividing by $a-1$,

$$x^4+\frac{4a}{a-1}x^3+\frac{6a}{a-1}x^2+\frac{4a}{a-1}x+1=0,$$

a reciprocal equation which is easily reduced by dividing by x^2, thus

$$[x^2+(1/x^2)]+\frac{4a}{a-1}[x+(1/x)]+\frac{6a}{a-1}=0,$$

Putting $x+(1/x)=y$, we shall have

$$y^2+\frac{4a}{a-1}y=-\frac{4a+2}{a-1}, \text{ whence } y=-\frac{2a}{a-1}\pm\frac{1}{a-1}\sqrt{2(a+1)},$$

and then

$$x=-\frac{a}{a-1}\pm\frac{1}{2(a-1)}\sqrt{2(a+1)}\pm\frac{1}{2(a-1)}\sqrt{2(2a^2+a+1)\mp4a\sqrt{2(a+1)}}.$$

Also solved by *A. H. BELL.*

GEOMETRY.

84. Proposed by FREDERICK R. HONEY, Ph. B., Instructor in Mathematics in Trinity College, New Haven, Conn.

Find the locus of a point which will trisect all arcs having a common chord.

I. Solution by G. B. M. ZERR, A. M., Ph. D., President and Professor of Mathematics, The Russell College, Lebanon, Va.

In what follows the arc of the circle is assumed.

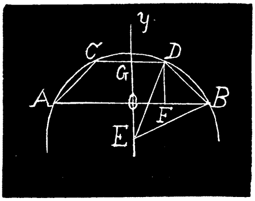
Let O be the origin, $EO=z$, $AB=2c$, $AC=CD=DB=n$.

Then $BF=\frac{1}{2}(2c-n)$, $OF=\frac{1}{2}n$, $DF=\sqrt{n^2-\frac{1}{4}(2c-n)^2}$, $EB^2=c^2+z^2$, $ED^2=EB^2=c^2+z^2$, $EG^2=c^2+z^2-\frac{1}{4}n^2$.

EG=also $EO+DF=z+\frac{1}{2}\sqrt{4n^2-(2c-n)^2}$.

$\therefore\ c^2+z^2-\frac{1}{4}n^2=z^2+z\sqrt{4n^2-(2c-n)^2}+n^2-c^2+cn-\frac{1}{4}n^2$,

or $2c^2-(n^2+cn)=z\sqrt{3n^2-4c^2+4cn}$ ....(1).



Equation to circle is $x^2+(y+z)^2=c^2+z^2$,

or $x^2+y^2+2yz=c^2$. $\therefore\ z=(c^2-x^2-y^2)/2y$ .....(2).

$OF^2+FD^2=x^2+y^2=\frac{1}{4}n^2+n^2-\frac{1}{4}(2c-n)^2$.

$\therefore\ n^2+cn=x^2+y^2+c^2$.

$\therefore\ n=\frac{1}{2}(\sqrt{4x^2+4y^2+5c^2}-c)$ ....(3).

(2) and (3) in (1) gives

$$2c^2-x^2-y^2-c^2=\frac{c^2-x^2-y^2}{2y}\sqrt{4x^2+4y^2-\frac{1}{4}(\sqrt{4x^2+4y^2+5c^2}-c)^2}.$$

$$\therefore\ 4y^2=3x^2+3y^2-\tfrac{3}{2}c^2+\tfrac{1}{2}c\sqrt{4x^2+4y^2+5c^2},$$

$$(2y^2-6x^2+3c^2)^2=c^2(4x^2+4y^2+5c^2).$$

$$(y^2-3x^2)^2+2c^2(y^2-5x^2)+c^4=0.$$

II. Solution by the PROPOSER.

Let *abcd*—Fig. 1—be any arc, and *ad* the chord. Bisect *ad* at *o*, and draw *eof* perpendicular to it. We wish to find the locus of a point *c* whose distance from a given straight line *ef* is one half the distance from a given point *d*.

In order to write the equation of this curve, refer it to the coördinate axes *ad* (axis of X) and *ef* (axis of Y), intersecting at the origin *o*.

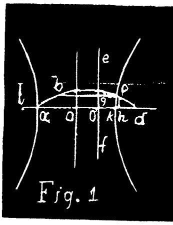


Fig. 1

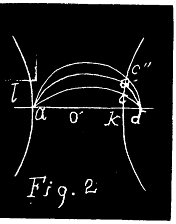


Fig. 2

Let $gc=x$. Therefore, from the definition $cd=2x$. Let $od=D$. $\therefore\ hd=D-x$.

Let $ch=y$. $\therefore\ (2x)^2=y^2+(D-x)^2$ or $4x^2=y^2+D^2-2Dx+x^2$.

$\therefore\ y^2-3x^2+D^2-2Dx=0$....(I). This is the equation of an hyperbola whose center is on the axis of abscissas. In order to determine the position of the center, eliminate the x term, and find the distance from the origin o to a new origin o'. Let E=distance from o to o'. $\therefore\ x=x'+E$. Substituting this value of x in equation (I), $y^2-3(x'+E)^2+D^2-2D(x'+E)=0$; or $y^2-3x'^2-6Ex'-3E^2+D^2-2Dx'-2DE=0$......(II). In this equation the x' terms should disappear.

$\therefore\ 6Ex'-2Dx'=0$. $\therefore\ E=-\frac{1}{3}D$.

That is, the distance from the origin o to the new origin, or the center of the hyperbola, o', is equal to one third of the distance from o to d; and the minus sign indicates that the measurement should be laid off to the left of the origin o. Substituting this value of E in equation II, and omitting accents, we have $y^2-3x^2+2x-\frac{1}{3}D^2+D^2-2x+\frac{2}{3}(D^2)=0$.

$\therefore y^2-3x^2=-\frac{1}{3}(4D^2)$. This is the equation of an hyperbola referred to its center o' as the origin. To write it in the ordinary form, that is, in terms of the transverse and conjugate axes, multiply each term by C; *i. e.* let $\sqrt{C}=$semi-transverse axis. Thus $Cy^2-3Cx^2=-\frac{1}{3}(4CD^2)$.

When in this form the product of the coefficients of the x^2 and y^2 terms should be equal to the remaining term. That is $-3C^2=-\frac{1}{3}(4CD^2)$......(III). $\therefore C=\frac{1}{9}(4D^2)$, and equation III becomes $\frac{1}{9}(4D^2)y^2-\frac{1}{3}(4D^2)x^2=-\frac{1}{27}(16D^4)$.

The semi-transverse axis$=\sqrt{[\frac{1}{9}(4D^2)]}=\frac{1}{3}(2D)$. The semi-conjugate axis $=\sqrt{[\frac{1}{3}(4D^2)]}=2D/\sqrt{3}$.

Since the distance from the center of the curve to either focus is equal to the square root of the sum of the squares of the semi-axes, the distance from o' to either focus$=\sqrt{\{[\frac{1}{9}(4D^2)]+[\frac{1}{3}(5D^2)]\}}=\frac{1}{3}(4D)$. We can therefore make the following construction—Fig. 2. Draw ad the chord of the arc acd. Trisect ad at o' and k. Produce da to l, making $al=ao'=o'k=kd$. With ak as a transverse axis, and l and d as foci, construct the branch of the hyperbola $kcc'c''$, which will intersect all arcs having the common chord ad at c, c', c'', etc., making the arcs cd, $c'd$, $c''d$, respectively, equal to one-third of the arcs acd, $ac'd$, $ac''d$, etc.

CALCULUS.

67. Proposed by BENJ. F. YANNEY, A. M., Professor of Mathematics in Mount Union College, Alliance, O.

A man starts to walk at a uniform rate across a draw-bridge just as it begins to move. He walks the full length of the bridge and back, in the same time that it takes the bridge to make a half revolution. How for does he ride, the length of the bridge being 250 feet, and its velocity uniform about a center axis?

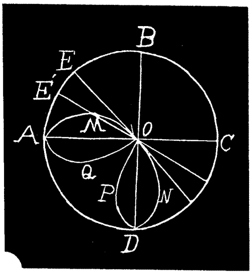
I. Solution by G. B. M. ZERR, A. M., Ph. D., President and Professor of Mathematics, The Russell College, Lebanon, Va.

In my solution of problem 45, I deduced the equation, $\rho=r(m-\theta)/m$. Let $m=\frac{1}{4}\pi$. $\therefore \rho=(r/\pi)(\pi-4\theta)$ is the equation to the man's path in space.

$$\therefore S=\frac{2r}{\pi}\int_0^{\frac{1}{2}\pi}\sqrt{16+(\pi-4\theta)^2}\,d\theta=\tfrac{1}{2}r\sqrt{16+\pi^2}+\frac{8r}{\pi}\left(\frac{\pi+\sqrt{(16+\pi^2)}}{4}\right)$$

$$=\tfrac{125}{2}\sqrt{16+\pi^2}+\frac{1000}{\pi}\log\frac{\pi+\sqrt{(16+\pi^2)}}{4}. \quad \therefore S=547.468 \text{ feet.}$$

II. Solution by J. SCHEFFER, A. M., Hagerstown, Md.



According to the conditions of the problem, A being the starting point of the man, he will be at O, going in the curve AMO, when the draw-bridge has turned through $\angle AOE=45°$; when the latter has turned through $\angle AOB=90°$, the man is in D, having passed through the curve OND; after the bridge has turned through $\angle AOF=135°$, the man is at O again, having moved through the curve DPO, and after the